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POSSIBLE NEW EXPERIMENTAL TEST OF GENERAL RELATIVITY THEORY*

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In a paper now in process of publication,[1] it is argued that only the planetary orbit precession provides real support for the full structure of the general theory of relativity. The other two of the three "crucial tests," the gravitational red shift and deflection of light, can be inferred correctly from the equivalence principle and the special theory of relativity, both of which are well established by other experimental evidence. It is also pointed out that a terrestrial or satellite experiment that would really test general relativity theory would have either to use particles of finite rest mass in such a way that the equation of motion can be confirmed beyond the Newtonian approximation, or to verify the second-order deviations of the metric tensor from its Minkowski form.

In an attempt to devise a feasible experiment that might accomplish one of these objectives, we have calculated the properties of a spinning test particle (torque-free gyroscope). We start from the covariant equations of Papapetrou[2] for the motion of the center of mass and the spin angular momentum, generalized by inclusion of a nongravitational constraining force $\vec{F}$, and work to lowest order. The motion of the center of mass in the gravitational field of the rotating earth is then described by the Newtonian equation

$$m(d\vec{v}/dt) = -(GmM/r^3)\vec{r} + \vec{F}, \tag{1}$$

where m is the rest mass of the particle, $\vec{r}$ is its coordinate, $\vec{v} = d\vec{r}/dt$ is its velocity, G is the Newtonian gravitational constant, and M is the mass of the earth. The spin angular momentum vector measured by a co-moving observer, $\vec{S}^0$, obeys the equation

$$d\vec{S}^0/dt = \vec{\Omega} \times \vec{S}^0, \tag{2}$$

where

$$\vec{\Omega} = (\vec{F} \times \vec{v})/2mc^2 + (3GM/2c^2r^3)(\vec{r} \times \vec{v}) + (GI/c^2r^3)[3(\vec{\omega}\cdot\vec{r})\vec{r}/r^2 - \vec{\omega}]; \tag{3}$$

$I = 2MR^2/5$ is the moment of inertia of the earth of radius R, assumed to be homogeneous, and $\vec{\omega}$ is its angular velocity vector. The first term on the right side of (3) is the Thomas precession,[3] which is a special relativity effect. The other two are the lowest order effects of general relativity; the second term arises whether or not the earth is rotating, and the third term is the earth rotation effect of Lense and Thirring.[4] While the second term involves the first-order deviations of the metric tensor from its Minkowski form, which can be calculated without the use of general relativity,[1] it also depends on the equation of motion of matter of finite rest mass beyond the Newtonian approximation. It is therefore a genuine consequence of general relativity. The same is true of the third term, since in addition it depends on off-diagonal space-time components of the metric tensor.

Equations (2) and (3) may be obtained either from the standard or the isotropic form of the Schwarzschild line element, and using for the supplementary condition on the angular momentum tensor either that of Corinaldesi and Papapetrou[5] or of Pirani.[6] The equation of motion of the spin in the nonrotating, earth-centered coordinate system looks quite different in these four cases, but they all agree when expressed in terms of the spin measured by a co-moving observer. It should also be remarked that the corrections to Eq. (1) that arise from the spin are unobservably small in any realizable situation.

It follows at once from the form of Eq. (2) that the magnitude of the spin angular momentum measured by a co-moving observer is constant in time. Thus if the moment of inertia of the spinning particle does not change, the angular velocity of rotation is constant, and the spinning particle behaves like a clock which can be set to any desired frequency. This frequency exhibits Doppler and gravitational shifts when observed from outside, just like that of a more conventional clock. It is possible that its frequency stability could be made to compare favorably with those of other types of precision clocks. It also follows from (2) that a number of spinning particles with various magnitudes and directions for their angular momentum vectors maintain fixed angles of these vectors with respect to each other. The vector $\vec{\Omega}$, which in general is not constant, is their common angular velocity of precession with respect to the external "fixed stars"; in comparing their directions with the outside world, a correction must of course be made for aberration whenever $\vec{v} \neq 0$.

If a spinning particle is in free fall, as in a satellite, then $\vec{F}=0$. For an orbit in the earth's equatorial plane, for example,

$$\vec{\Omega}=(3GM/2c^2r)\vec{\omega}_0-(2MGR^2/5c^2r^3)\vec{\omega}, \tag{4}$$

where $\vec{\omega}_0=(\vec{r}\times\vec{v})/r^2$ is the instantaneous orbital angular velocity vector of the particle. The minus sign in Eq. (4) deserves some comment. The third term of Eq. (3) tends to cause a spinning particle to precess in the same direction as the rotating earth at the poles ($\vec{r}$ parallel or antiparallel to $\vec{\omega}$), but in the opposite direction at the equator ($\vec{r}$ perpendicular to $\vec{\omega}$). This is physically reasonable if we think of the moving earth as "dragging" the metric with it to some extent. At the poles, this tends to drag the spin around in the same direction as the rotation of the earth. But at the equator, since the gravitational field falls off with increasing r, the side of the spinning particle nearest the earth is dragged more than the side away from the earth, so that the spin precesses in the opposite direction.

If the center of mass of the spinning particle is constrained to remain at rest with respect to the rotating earth, as in an earth-bound laboratory,[7] then

$$\vec{v}=\vec{\omega}\times\vec{r}, \quad d\vec{v}/dt=\vec{\omega}\times\vec{v}. \tag{5}$$

The required constraining force $\vec{F}$ can then be found from (1) and (5), and substituted into (3). When the particle is at the surface of the earth at latitude λ, the precession angular velocity may be written in the form

$$\vec{\Omega}=[(4gR/5c^2)(1+\cos^2\lambda)-(\omega^2R^2/2c^2)\cos^2\lambda]\vec{\omega} + (4g\sin\lambda/5\omega c^2)(\vec{\omega}\times\vec{v}), \tag{6}$$

where $g=GM/R^2$ is the acceleration of gravity at the surface of the earth. Only the square bracket term in Eq. (6) gives rise to a secular precession of the spin axis, and the second part of it is very small compared to the first. Thus to good approximation, a particle with spin axis perpendicular to the earth's axis precesses at the rate $2\pi(4gR/5c^2)(1+\cos^2\lambda)=3.5\times10^{-9}(1+\cos^2\lambda)$ radians per day. It also follows from Eq. (3) that the corresponding effects caused by the sun and moon are negligibly small in comparison.

A secular precession of 6×10^{-9} radian per day would be very difficult, but perhaps not impossible, to observe. Professor W. M. Fairbank and Professor W. A. Little of this department are exploring the possibility of using for this purpose a gyroscope that consists of a superconducting sphere supported by a static magnetic field.[8] Such a gyroscope would also be of interest as a device for performing experiments in low-temperature physics. If it could be made to operate exceedingly well, it might in addition be used for an experimental test of Mach's principle, by comparing the orientation of its axis with a field of "fixed stars" over a period of a year or so. Most of the experimental difficulties that seem to arise with a high-precision gyroscope are greatly reduced if the gyroscope does not have to be supported against gravity. This, together with the fact that ω_0 is generally much larger than ω, suggests that experiments of this type might be more easily performed in a satellite than in an earth-bound laboratory.

A full account of this work will be submitted shortly for publication in the Proceedings of the National Academy of Sciences.

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